

Large N reduction with the Twisted Eguchi-Kawai model

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ABSTRACT: We examine the breaking of Z_N symmetry recently reported for the Twisted Eguchi-Kawai model (TEK). We analyse the origin of this behaviour and propose simple modifications of twist and lattice action that could avoid the problem. Our results show no sign of symmetry breaking and allow us to obtain values of the large N infinite volume string tension in agreement with extrapolations from results based upon straightforward methods.

KEYWORDS: large N , Yang-Mills theory.

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1. Introduction

The large N limit of gauge theories continues to be a fascinating source of simplifications of the complicated field theory dynamics, which has yet defied our capacity for finding an analytic solution. A puzzling old phenomenon is space-time reduction, namely the possibility put forward by Eguchi and Kawai [1] that the Physics of the large N limit would be independent of the physical spatial volume. The idea originated by comparing the loop equations [2] of a lattice gauge theory on a d -dimensional torus of different sizes. The corresponding equations were independent of the lattice size provided the Z_N^d symmetry of the infinite volume d -dimensional theory is preserved by the finite volume theory. Taking the idea to the extreme, a one-point periodic lattice Matrix model (the EK model) would encompass the dynamics of infinite volume. Although, the idea seems to work at strong coupling, it was observed that the symmetry and the reduction is lost for weak coupling [3] in four dimensions. The same authors proposed a way out, called Quenched Eguchi-Kawai model (QEK), in which the problem could be avoided by freezing in the eigenvalues of the link matrices. The present authors [4]-[5] realized that the weak coupling behaviour of the reduced model depends strongly on the boundary conditions while the original Eguchi-Kawai proof does not. They proposed imposing twisted boundary conditions in such a way as to guarantee that the symmetry would be respected at weak coupling. The realization of this idea, known as the twisted Eguchi-Kawai model (TEK), was studied analytically and numerically and passed all tests with flying colours. Being more effective in approaching the large N limit than the QEK model, it was used both numerically and analytically for a better understanding of large N gauge theory [6].

It is interesting to recall how the TEK model is capable of recovering the perturbative behaviour of the infinite volume theory. Indeed, by expanding around the minimum action solutions, in a suitable $SU(N)$ Lie algebra basis, one obtains very similar Feynman rules as for a $L^4 \equiv (\sqrt{N})^4$ lattice gauge theory. The momentum degrees of freedom follow from the ones in the group. The propagators are exactly the same as in the ordinary L^4 lattice theory, but the vertices reflect the non-commutative nature of the group in that, in addition to

momentum conservation, there are momentum dependent phase factors. These phases are basically the structure constants of the group written in this basis. In Ref. [5] the present authors showed how phases cancel each other out for planar diagrams, while they remain for non-planar ones. Thus, all one needs to recover the perturbative behaviour of large N gauge theory is that the phases kill the contribution of the corresponding diagrams at large N . This was plausible since the phases oscillate more and more as we keep the momenta fixed and take the large N limit. Translating the mechanism to the continuum [7] led to non-commutative momenta and Feynman rules which coincide with what later came to be known as non-commutative quantum field theory [8]. Proof of cancellation of the phases goes unchanged. The connection with non-commutative quantum field theory triggered a revival in the interest upon the TEK model, as a possible regulator of the non-commutative gauge theory [9].

Recently, however, new results [10]-[11]-[12] have shown that as N is increased to higher values (≥ 100) the TEK model shows a pattern of symmetry breaking at intermediate couplings, with pronounced hysteresis cycles. Although, the continuum limit of the theory sits at weak coupling, where symmetry is restored, the phase transition point β_c seems to move to higher and higher values of the coupling as N grows. Since only loops of size smaller than L can be computed, the authors of Ref. [12] claimed that their physical size $La(\beta_c)$ shrinks to zero in the large N limit, leaving no window for a continuum reduction. In parallel, evidence was given [13] that the QEK model also displays symmetry breaking at weak coupling.

We refer that, on the positive side, there have been important progress recently in extending the reduction idea to gauge theories with fermions in the adjoint representation [14], where the fermions help in stabilising the symmetry. Furthermore, a new approach to reduction has been initiated by Narayanan, and Neuberger [15] in which by keeping both space and group degrees of freedom one can approach the large N limit with optimal use of the degrees of freedom.

The purpose of this paper is to analyse the origin and nature of the symmetry breaking on the TEK model and examine ways in which the reduction idea can be restored for pure Yang-Mills gauge theory.

2. Weak-coupling Analysis

The TEK model is a model of d $SU(N)$ random matrices fluctuating with a probability distribution derived from the action

$$S = bN \sum_{\mu\nu} (N - z_{\mu\nu} \text{Tr}(P_{\mu\nu})) \quad (2.1)$$

where the z factors are elements of \mathbf{Z}_N

$$z_{\mu\nu} = \exp\{2\pi i \frac{n_{\mu\nu}}{N}\} \quad (2.2)$$

and

$$P_{\mu\nu} = U_\mu U_\nu U_\mu^\dagger U_\nu^\dagger \quad (2.3)$$

This model follows from considering $SU(N)$ lattice gauge theory (with Wilson action) in a periodic lattice with twisted boundary conditions and collapsing the lattice to a single point. According to the proof of reduction[1], this model should be equivalent to the infinite volume lattice gauge theory at large N . This should be so, irrespective on the value of $z_{\mu\nu}$. However, the proof of *reduction* relies on the hypothesis that the reduced model respects the Z_N^d symmetry of the large volume theory. The action is obviously symmetric under

$$U_\mu \longrightarrow e^{2\pi i p_\mu / N} U_\mu \quad (2.4)$$

but this symmetry can be broken spontaneously, as we know this is actually the case [3] for the original EK model ($z_{\mu\nu} = 1$) in 4 dimensions. From now on we will restrict ourselves for simplicity to the four-dimensional case ($d = 4$).

There is a special interest in focusing upon the weak-coupling region for two main reasons. The first, is that customarily the origin of spontaneous symmetry breaking is the fact that the different classical vacua (the absolute minima of the potential) are not invariant under the symmetry. This is actually the case for the EK model, and explains why it fails to achieve reduction. The second reason, is that the continuum limit of the theory lies at zero-coupling. Thus, the only way in which we can dream of maintaining the reduction idea in the continuum is by preserving the symmetry in the weak-coupling region.

These considerations motivated us to examine the situation at weak coupling and to realize that the choice of $z_{\mu\nu}$ has a dramatic impact upon the classical vacua and symmetry breaking [4]-[5]. The minimum action solution (called *twist-eaters*) $U_\mu = \Gamma_\mu$ can be made invariant under a combined transformation Eq. 2.4 and a gauge rotation. It is impossible though to preserve the full Z_N^4 of transformations. At most one can preserve a Z_N^2 subgroup. Guided by the requirement of treating all directions equally, we proposed to take $N = L^2$ and the following *symmetric twist* choice of the antisymmetric twist tensor:

$$n_{\mu\nu} = kL \quad \text{for } \mu < \nu \quad (2.5)$$

Our particular proposal was taking $k = 1$, but indeed any integer which is coprime with L suffices ¹. This twist tensor is by no means unique, but it is the most symmetric proposal, and the remaining symmetry group can be written as Z_L^4 and allows the fulfilment of the reduction concept as L goes to infinity. Indeed, all numerical studies of the TEK model done at the time of our paper revealed that the symmetry was preserved at all values of the coupling.

Since the symmetry cannot be broken to any order in perturbation theory, the perturbative expansion of the TEK model must reproduce the one of infinite volume gauge theory at infinite L . The way in which this is achieved is fascinating, and anticipated by many years developments in non-commutative space-time field theories (for a review consult Ref. [8]). Let us review here some aspects, generalising the formulas of Ref. [5] to arbitrary k coprime with L .

¹values of k other than one were considered in the non-commutative geometry context and also in Ref. [10].

The first step is to expand the link variables around twist-eaters: $U_\mu = e^{iA_\mu}\Gamma_\mu$ and replace this expression into $P_{\mu\nu}$:

$$z_{\mu\nu}P_{\mu\nu} = e^{iA_\mu}e^{i\delta_\mu A_\nu}e^{-i\delta_\nu A_\mu}e^{-iA_\nu} = e^{iG_{\mu\nu}} \quad (2.6)$$

where

$$\delta_\mu\phi \equiv \Gamma_\mu\phi\Gamma_\mu^\dagger \equiv D_\mu\phi + \phi \quad (2.7)$$

Obviously $G_{\mu\nu}$ is hermitian and given by the Baker-Campbell-Hausdorff formula:

$$G_{\mu\nu} = D_\mu A_\nu - D_\nu A_\mu + O(A^2)$$

Our notation is chosen to stress the resemblance with the ordinary field-strength tensor. And indeed the similarity goes beyond, since the operators D_μ turn out to commute among themselves and have the same spectrum as the ordinary lattice derivatives in an L^4 lattice:

$$D_\mu\lambda(q) = (e^{iq_\mu} - 1)\lambda(q) \quad (2.8)$$

with q_μ and integer multiple of $2\pi/L$. The $SU(N)$ matrices $\lambda(q)$, whose explicit form we will not need, are the corresponding eigenvectors. Hence, perturbation theory propagators will reproduce the standard lattice propagator in an L dimensional lattice. This gives us information about the nature of the finite L corrections to the reduction picture. Of course, before reaching that step, one must handle zero-modes of the quadratic form in A_μ . However, since the vacuum is unique up to gauge transformations, one can get rid of them by an appropriate gauge fixing. Gauge transformations adopt the familiar form

$$e^{iA_\mu} \longrightarrow \Omega e^{iA_\mu} (\delta_\mu\Omega)^\dagger \quad (2.9)$$

if we conceive δ_μ as the operator of displacement by one lattice spacing in the μ direction.

However, the reduced model at finite L is not simply equivalent to the ordinary lattice gauge model in a finite volume. The vertices of the theory contain (in addition to momentum conservation delta functions) momentum dependent phases, which (the anti-symmetric part) are simply the values of the structure constants of the theory in the basis of eigenstates of D_μ :

$$\text{Tr}(\lambda(q)\lambda(p)\lambda(k)) = \delta(q+p+k)e^{i\Phi(q,p)\theta_L} \quad (2.10)$$

with $\Phi(q,p) = -\langle q,q \rangle - \langle p,p \rangle - \langle p,q \rangle$, up to additional terms depending on the normalization of the $\lambda(q)$ matrices. The symbol $\langle p,q \rangle$ stands for the quadratic form

$$\langle p,q \rangle = p_0(q_1 - q_2 + q_3) + p_1(q_2 - q_3) + p_2q_3$$

and

$$\theta_L = \frac{L\bar{k}}{2\pi} \quad (2.11)$$

where \bar{k} is an integer, coprime with L , and defined through the relation:

$$k\bar{k} = mL + 1 \quad (2.12)$$

In fact, \bar{k} and θ_L are the only places in which the perturbative series depends on the choice of k . The momentum dependent phases are crucial to recover the large N dynamics since, as proven in Ref [5]-[16], the phase factors from different vertices cancel out for planar diagrams. For non-planar diagrams, the remaining phase factor will oscillate more and more strongly as L grows and so, we argued, it will cancel it out.

Some comments are in order. First, that we are implicitly assuming that the large N limit is taken on the lattice theory at fixed b , so that one does not have to worry about ultraviolet divergences. Only later, the continuum limit is taken by the customary scaling as b goes to infinity, with the large N beta function. A different story occurs if we want to take a *double scaling limit* in which we keep the continuum non-commutativity parameter fixed $\theta = \theta_L a^2(\beta)$, to obtain continuum non-commutative theories. Although, very interesting by itself, we will not bother about it in the present work, since it is not our main goal.

Altogether, we have reviewed our proof of reduction based on perturbation theory, which serves to complement the loop equation proof of Eguchi and Kawai. Focusing again upon k dependence, notice that the suppression of non-planar diagrams is bigger for large values of θ_L and hence, of \bar{k} .

This panorama remained stable for many years, until recent simulations done at larger values of N showed a pattern of Z_L^4 symmetry breaking at intermediate couplings [10]-[11]-[12]. As a matter of fact, the authors of Ref. [11] express doubts that the reduction applies for physical sizes below $1/T_c$, and in Ref. [12] the conclusion seems to be that the window of physical sizes where the symmetry is preserved shrinks to zero as N grows. Motivated by these results we decided to explore the situation in more detail.

The authors of Ref. [11] suggest that the origin of the symmetry breaking could be due to other extrema of the TEK action functional [17]. We follow the nomenclature used in those references and refer to these extrema as *fluxons*. These extrema are given by $U_\mu = \Gamma'_\mu$ such that the plaquette values are Z_N elements:

$$\Gamma'_\mu \Gamma'_\nu = e^{-\frac{2\pi i n'_{\mu\nu}}{N}} \Gamma'_\nu \Gamma'_\mu \quad (2.13)$$

Each fluxon has its characteristic pattern of symmetry breaking and a set of products having possible non-vanishing traces. We call these products *open paths* for obvious reasons. One particular example of fluxon are the twist-eaters themselves ($n'_{\mu\nu} = n_{\mu\nu}$). For them the open paths must have length proportional to L in all directions. The extreme opposite case is that of singular *torons* (using the terminology of Ref. [18]) having $U_\mu = z_\mu \mathbf{I}$. In that case, the symmetry breaks down completely and all paths are open.

As a matter of fact, fluxons are extrema of the TEK model for any choice of twist $n_{\mu\nu}$. This is a huge number of extrema. What the choice of twist determines is the action of each fluxon. However, as argued in Ref. [11], as b decreases, entropy might well overcome the energy difference and the system can jump to the vicinity of a different fluxon, inducing symmetry breaking. Let us examine the value of the action difference between a singular toron and a twist eater:

$$\Delta S = 24bN^2 \sin^2\left(\frac{\pi k}{L}\right) \quad (2.14)$$

As we take the large N limit with b and k fixed, this difference in action grows as N . This, might not be enough if the entropy difference is order N^2 (as is natural since that is the number of degrees of freedom).

The aforementioned problem seems to have a simple solution. One has to take values of k which are order L . Is this possible? A priori there seems to be no objection, since the perturbative proof of reduction only requires k to be coprime with L . The only possible effect at finite L was hidden in the value of θ_L . We will examine this point later.

It is clear, from Eq. 2.14, that the maximum action difference that we can obtain by changing k is $12bN^2$. Is this enough? If the entropy difference is proportional to N^2 then there would be a weak coupling region in which symmetry would be respected and a continuum limit taken. The question then would be: What is then the minimum value of b which can be explored?

Unfortunately, there is even a more worrisome situation if the entropy difference between the twist-eater and the singular toron grows as $N^2 \log(N)$, as suggested by the analysis of Ref. [18]. The log behaviour comes from the vanishing of the linear term in $G_{\mu\nu}$. Fluctuations are then quartic rather than quadratic.

Nevertheless, if we study fluctuations around the singular toron, we obtain

$$S = 24bN^2 \sin^2\left(\frac{\pi k}{L}\right) + 6bN \cos\left(\frac{2\pi k}{L}\right) \text{Tr}(G_{\mu\nu}^2) + \dots \quad (2.15)$$

Thus, we see that for $k/L > 1/4$, the singular toron becomes unstable (it becomes a local maximum). This fact applies as well to all other torons.

What about other fluxons? Could we fall into the same problem with other fluxons? First of all, one must say that the singular toron is the most dangerous case, because it has the largest number of quartic fluctuations. In no other case can one have $N^2 \log N$ contributions. There are cases, nonetheless, in which $N \log N$ contributions can appear. For that reason we have analysed all fluxons to see if we could get fluxons having action differences of order one, and possessing short open paths. Indeed, this possibility cannot occur. Furthermore, if k is order L the bounds get stronger and one can rule out the possibility that an $N \log N$ entropy could win over the energy². The detailed analysis is lengthy and will be presented elsewhere[19].

Hence, we conclude that a judicious choice of $k(L)$ could solve the instabilities observed in the TEK model for $k = 1$ and lead to a reduced large N limit. Our arguments are based on weak coupling analysis and also explain why the $k = 1$ failed. These arguments cannot rule out differences of entropy of order N^2 , but as mentioned earlier this will leave a window of b values where reduction will apply. This can be important as a matter of principle even though the values of b for large N reduction to apply could be too large for all practical purposes. For that reason, it is important to realize that all the dangerous fluxons are not features of the continuum theory. Thus, their action can be changed appropriately by changing the lattice action of the TEK model, without affecting the continuum formulation in the symmetry breaking phase. This strategy is currently being studied by the present authors.

²To simplify the proof we assumed that L is prime and \bar{k} is not of order L .

Finally, we should comment, that for a faster approach to the large N infinite volume theory a large value of \bar{k} is desirable. Indeed, the value of \bar{k} can also be chosen to grow proportionally to L . To show that all these conditions are not mutually exclusive, we give an example: $k = \bar{k} = (4p - 1)$ and $L = (16p - 8)$, for any positive integer p .

3. Numerical results

Motivated by our previous arguments, we studied the behaviour of the TEK model for different values of k by Monte Carlo methods. We employed the Fabricius-Hahn heat-bath algorithm [20]. Our first goal was to determine the value of $b_c(L, k)$ below which symmetry breaking is observed when starting from cold initial configurations based on twist-eaters. Since large hysteresis have been reported, $b_c(L, k)$ would actually turn out to be smaller than the phase transition point. But for our purposes it is admissible to do simulations in a metastable vacuum provided the lifetimes are large enough for accumulating sufficient statistics. The $k = 1$ case has been studied previously by other authors[10]-[11]-[12], but we have extended the analysis to higher values of L . It is known that for $L \leq 9$ there is no evidence for symmetry breaking. Beyond, the traces of the link variables develop a non-zero value for $b < b_c(L, 1)$. The values of $b_c(L, 1)$ are plotted as a function of L in Fig. 1. For presentation purposes we have omitted from the Figure our largest group values: $b_c(23, 1) = 0.8075(25)$ and $b_c(27, 1) = 1.1925(25)$.

The $k = 2$ case was also addressed in Ref. [10], where it was found that it also exhibits symmetry breaking for $L \geq 19$. Here we have extended this analysis to much higher values of L . The values of $b_c(L, 2)$ seem to match with those of $k = 1$ if plotted as a function of L/k as seen in Fig. 1. Indeed, there is small downward shift of 0.0085, in addition to scaling, which we have incorporated in the plot. Scaling in L/k (with the same shift) is also observed for $k = 3$ although the range is limited, since there is no breaking for $L \leq 28$. For $k = 4$ we verified that symmetry remains unbroken up to (and including) $L = 37$. For $L=39$ and $b=0.34$ we observed symmetry breaking, giving a values of $b_c(L, 4)$ which is consistent with scaling. For $k = 5, 6, 7$ we have been unable to observe symmetry breaking. All our results point towards a symmetry breaking curve $b_c(L, k)$ which scales approximately with L/k and is defined only for $L/k > 9 - 9.5$. To show evidence of the symmetry restoration, we show in Fig. 2 the distribution of eigenvalues of the four link variables for one of $L = 23$, $k = 7$, $b = 0.37$ configurations, showing the characteristic structureless flat pattern.

Scaling is not surprising if the dominant breaking mechanism is through transitions to the singular torons, since their action divided by N^2 , is indeed a function of L/k . Actually, we found that a nice order parameter is the expectation value of the $\mu - \nu$ plaquettes formed by smeared link variables. Fluctuations are reduced considerably by smearing, and the mean value is centred at zero in the unbroken phases, while for $k = 1$ and $b < b(L, 1)$ moves to the position corresponding to torons. This is indeed, a confirmation that torons are dominating the path integral in that region. Scaling of $b_c(L, k)$ with L/k might lead to worries related to the possibility that traces of open paths of approximate length L/k acquire a non-zero value. If that was the case, the advantages of exploring higher values of b with higher values of k would be lost. Indeed, the fluctuations of traces of open paths

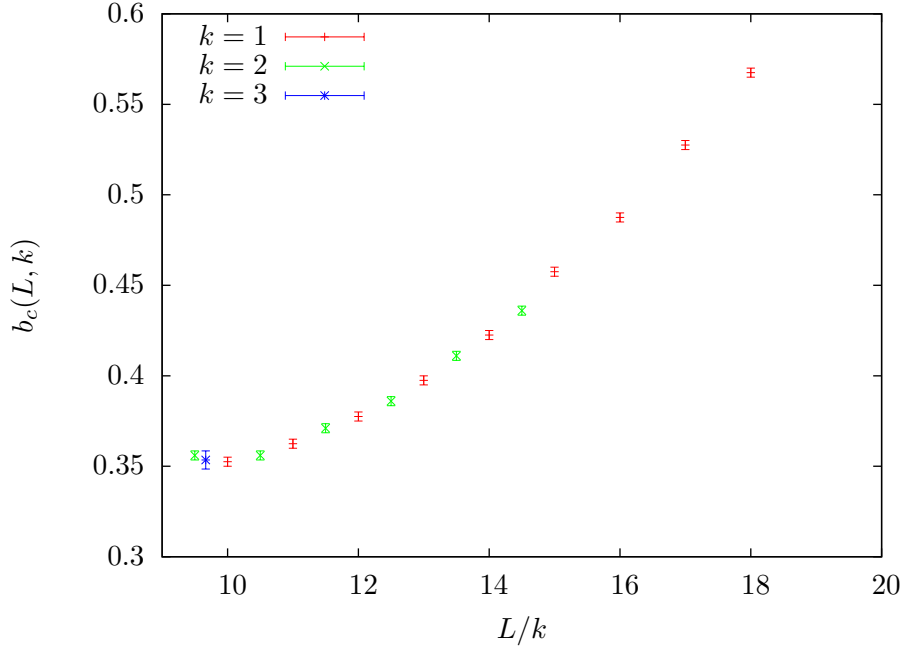


Figure 1: The values of $b_c(L, k)$ are plotted as a function of L/k . The values for $k = 2$ and $k = 3$ are shifted upwards by 0.0085 to highlight scaling. The values of $k = 1$ are from Ref.[12].

can be computed in perturbation theory and depend on the choice of k . To leading order one has

$$\frac{1}{N^2} \langle |\text{Tr}(U_0^{m_0} \dots U_3^{m_3})|^2 \rangle \propto \left(\sum_{\mu} \sin^2 \left(\frac{\pi \sum_{\rho} n_{\mu\rho} m_{\rho}}{N} \right) \right) \quad (3.1)$$

Notice that paths having lengths of order L/k have the largest fluctuations. The risk is that these fluctuations could end up destabilising the system and leading to symmetry breaking. To rule out this possibility we analysed with particular detail all traces having segments of length approximately equal to L/k . In all cases the values of all traces were seen to oscillate with larger or smaller amplitude around zero.

Thus, our numerical results confirm the reasoning based on our weak coupling analysis in what respects toron dominance, and allows us to define a window of good choices of k . Our results point to $k > L/9$ as the region of k values necessary to avoid symmetry breaking, as was first conjectured in Ref. [10].

At high k values, signs of symmetry breaking have been observed in the series of k values $k = (L - 1)/2$ with odd $L \geq 17$. In fact, at $L = 17$ and $k = 8$, Ishikawa and Okawa found large values of $\text{Tr}(U_{\mu}^2)$ in their unpublished work. We also found the same phenomena at $L = 19, 21$ and 23 . We, then, made a very long run at $L = 17$ and $k = 8$ of 20000 sweeps. Our results exhibit large autocorrelation times in which the values of $\text{Tr}(U_{\mu}^2)$ are changing, so we cannot conclude that we have symmetry breaking. Furthermore, the smeared plaquettes exclude that this is due to toron dominance. It is interesting to notice that the common feature of these cases is that $|\bar{k} \bmod L| = 2$ and considered previously in connection with the non-commutative field theories [9].

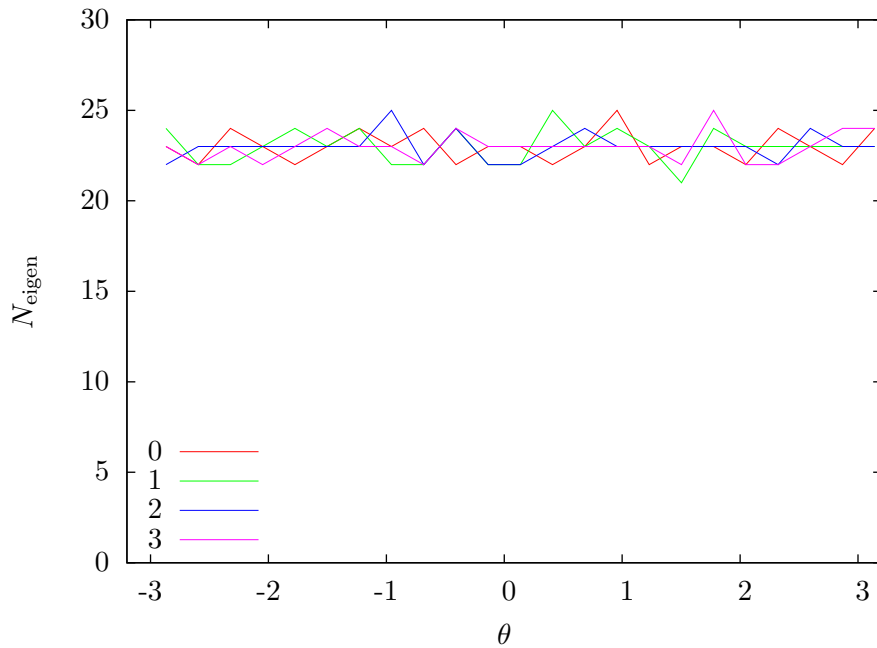


Figure 2: We display a histogram showing the distribution of angles from the eigenvalues $e^{i\theta}$ of the link matrices for a typical $L = 23$ $k = 7$ configuration at $b = 0.37$.

These observations, together with the lack of a formal proof that the symmetry will be respected at all values of the coupling and larger values of L , suggested us to adopt a more practical attitude, and ask ourselves if the TEK model can, at the present stage, provide useful results about the large N limit of Yang-Mills theory. The rest of this section will be devoted to showing that this is indeed the case.

Thus, we set ourselves to the task of exploring the model at the largest values of L at which our present computer resources allow us to collect enough statistics. We chose two values $L = 17$ and $L = 23$. This will enable us to quantify the L dependence of our results. In the first case, we chose $k = 5$ and in the second case $k = 7$, satisfying our preferred choice criteria. For the $L = 23$ case we generated 225 Monte Carlo configurations separated by 20 sweeps, after discarding the initial 1500 sweeps, for $b = 0.36$ and $b = 0.37$. A sweep is defined by one heat-bath update followed by five overrelaxation updates. Having two values of b is also useful, since it will allow us to analyse scaling of our results. At $L = 17$ we generated 1025 configurations at $b = 0.36$ with the same separation and discarded initial sweeps. First of all, notice that with our choice of b , we are exploring physical sizes much beyond $1/T_c$, avoiding the worries of Ref. [11]. One can use the estimates of $a(b = 0.36)T_c$ extracted from Ref. [21], to compute the value of $l_{\text{phys}}T_c$. For $L = 23$ and $b = 0.36$ this quantity is 2.75, well inside the confinement region.

Finally, we have attempted to extract physical results on the string tension from our data. This determination is an update of our pioneer study of string tension from reduced models [22]-[6]. Recently, other determinations based on the continuum reduction idea have appeared [23]. For that purpose, in each run we computed the expectation values of

Wilson loops of all sizes up to $\frac{(L-1)}{2} \times \frac{(L-1)}{2}$. The signal obtained for large loops is small and the results very noisy. To obtain a better signal to background ratio, we applied n_s smearing steps to our link variables and recomputed the loops with them. The smearing algorithm is

$$U'_\mu = \mathcal{U} \left[U_\mu + c \sum_{\nu \neq \mu} (z_{\nu\mu} U_\nu U_\mu U_\nu^\dagger + z_{\mu\nu} U_\nu^\dagger U_\mu U_\nu) \right] \quad (3.2)$$

where \mathcal{U} stands for the operator that projects onto unitary matrices. Most of our results were obtained for $c = 0.1$. For the $L = 17$ simulation we also used $c = 0.3$ and found that both values scale approximately with the variable fn_s of Ref. [24] ($f = 6c/(1+6c)$). Noise has dropped considerably at $fn_s = 4$.

In order to determine the string tension we computed Creutz ratios

$$\kappa(R, T) = -\log \left(\frac{W(R, T)W(R-1, T-1)}{W(R, T-1)W(R-1, T)} \right) \quad (3.3)$$

and we fitted those having $R, T \geq 4$ to a formula of the type:

$$\kappa(R, T) = \sigma - \gamma \left(\frac{1}{R(R-1)} + \frac{1}{T(T-1)} \right) \quad (3.4)$$

This dependence is obtained by plugging the leading corrections expected in perturbation theory for Wilson loops into the Creutz ratios formula. This method was used earlier, in a different context, with very good results [25]. Actually, it also works very well in our case. To show the quality of the fit, we display in Fig. 3 the value of $\sigma(R, T)$. The plotted quantity is defined as

$$\sigma(R, T) = \kappa(R, T) + \gamma \left(\frac{1}{R(R-1)} + \frac{1}{T(T-1)} \right) \quad (3.5)$$

obtained from different Creutz ratios after subtracting the correction proportional to γ obtained from the fit, as a function of the area RT . For a good fit, it should be flat, and its value provides an estimate of the string tension. This study has been done for all number of smearing steps with consistent results. As mentioned previously best fits are obtained for fn_s in the range 4–8. An example is given in Fig. 3 for the results at $b = 0.37$ and $L = 23$ after 15 smearing steps. It is peculiar that the values obtained for γ were of the same sign and similar magnitude to those predicted by the universal string theory behaviour [26]. To appreciate the dependence of our results upon smearing, we display in Fig. 4 the list of string tensions obtained as a function of the number of smearing steps for the $L = 23$, $b = 0.36$ case.

Finally, we collect our results in Table 1. We write the number of configurations, the value of b , k , \bar{k} and L , and the result for the string tension and plaquette expectation value. Statistical errors were estimated by a jack-knife method, and are of order 5%. A larger fraction of the error is systematic. The results depends on the detail of the fitting procedure (weight of the largest and smallest loops) and in the number of smearing steps. For the $L = 23$ case we have a wider range of loop sizes with reasonably small errors and

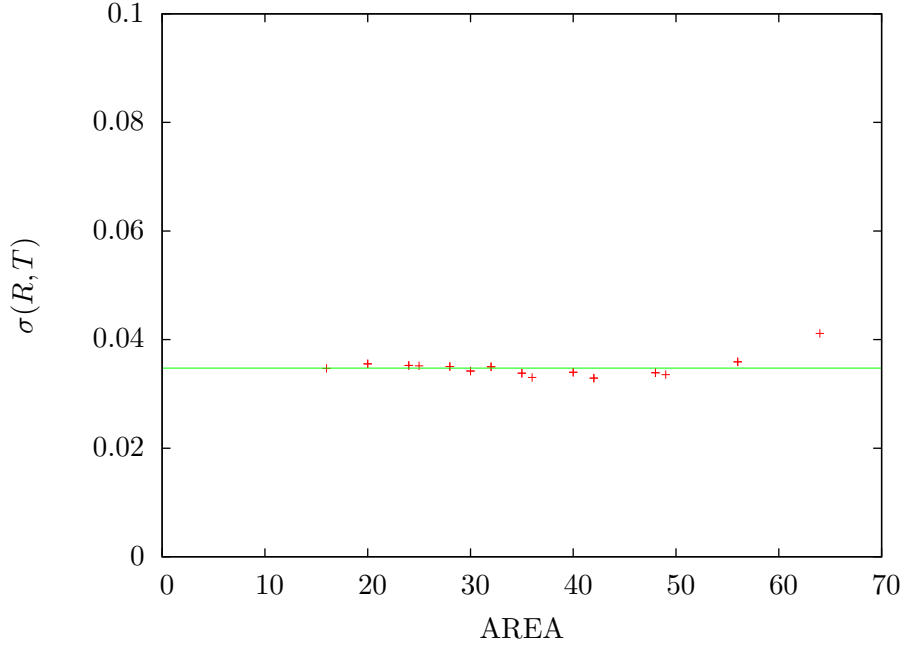


Figure 3: We show the quantity $\sigma(R, T)$ defined in Eq. 3.5 as a function of the area= $R \times T$. The horizontal line corresponds to the string tension obtained from the fit. Data are for the $L = 23$, $k = 7$, $b = 0.37$ configuration after 15 smearing steps.

N_{conf}	L	b	k	\bar{k}	σ	$\langle P_{\mu\nu} \rangle$
1025	17	0.36	5	7	0.055 (3)(6)	0.5581(1)
225	23	0.36	7	10	0.047 (3)(5)	0.5582(1)
225	23	0.37	7	10	0.035 (2)(4)	0.5789(1)

Table 1: The values of the parameters of our simulations together with the resulting value of the lattice string tension σ and the average value of the plaquette. The first error of the string tension is statistical and the second systematic.

the systematic errors are not bigger than 10%. For the $L = 17$ case the statistics is higher, but this is well compensated by the limited range of useful loop sizes. We also observed that at $\beta = 0.36$ the data presents large autocorrelation times. As emphasised before, our goal here is not to provide string tension determinations competitive with others in the market, but rather to illustrate that very reasonable results follow from the model. We draw the attention to how close our $L = 23$ results are from the values presented in Ref. [21] obtained from large N extrapolation from standard lattice gauge theory determinations at small values of N . The $L = 17$ results tend to be larger, but we find it premature, due to the problems mentioned previously, to draw any conclusions from this fact.

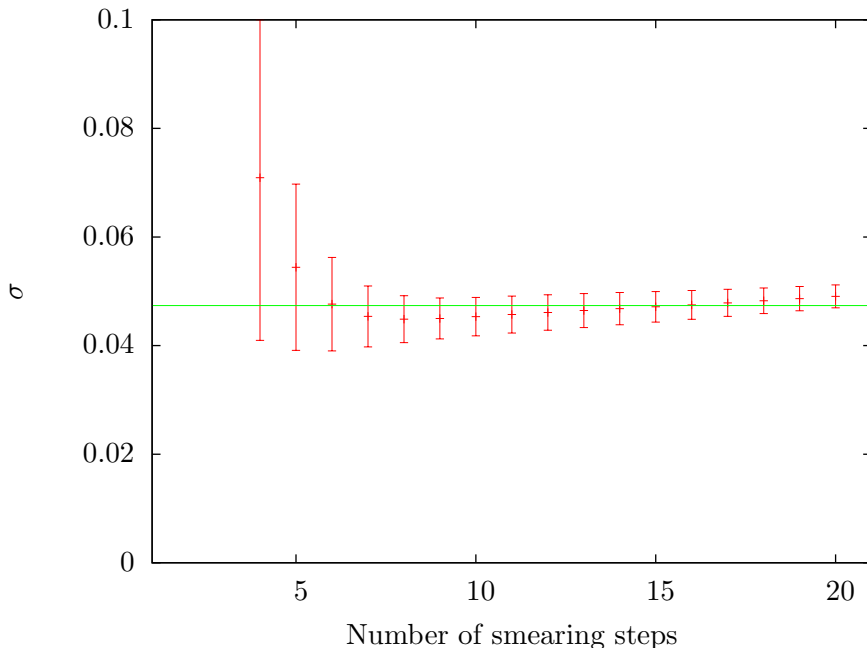


Figure 4: The different determinations of the string tension as a function of the number of smearing steps with $c = 0.1$ for $L = 23$ and $b = 0.36$. Errors reflect quality of the fit to Eq. 3.4.

4. Conclusions

In this paper we have analysed the Twisted Eguchi Kawai (TEK) model in the weak coupling phase for large group sizes $N = L^2$, and different values of the flux parameter k . Our result confirms that fluxons, and in particular torons(commuting fluxons), seem to be the main mechanism driving symmetry breaking for small k values. This suggests that taking k as a sufficiently large integer, and coprime with L , could solve the instability of the model without sacrificing the perturbative proof of reduction. Our numerical results confirm this expectation and show that for $L/k < 9$ no symmetry breaking is observed. Our best choice is actually $L/k < 4$ because in that case torons become unstable. It is also important to take into account that finite L effects coming from the underlying non-commutative field theory could be made smaller by taking large values of \bar{k} (defined in Eq. 2.12), even of order L .

It is clear that there is still a lot to understand about the dynamics of the TEK model at intermediate couplings. A full semiclassical analysis including fluctuations around fluxons is under way, and will be included in a future publication [19]. The possible evidence for a phase transition for $k = (L - 1)/2$ (at $L = 17, 19, 21, 23$) suggest that other fluxons, or a different mechanism could be at work. At this stage it is difficult to determine if there is any relation with the instabilities found in analytical [27] and numerical works [28] centred upon non-commutative field theories. It is obvious, however, that the twist-eater vacuum is the absolute minimum of the TEK model and cannot present “perturbative instabilities”, but these could be hints of other sort of problems observed numerically[28].

Lacking, at this stage, a proof that the symmetry breaking phase is not metastable and that reduction will survive the continuum limit, we decided to explore the present accessible ranges to see whether physics results could be extracted from it. We showed that one can access sizes which are much beyond $1/T_c$ and where confinement behaviour is patent. The values obtained for the string at our largest sizes $L = 23$ scale in the right fashion. Furthermore, they are surprisingly close to those obtained by extrapolation from small groups [21], given our limited statistics. The L dependence seems to be sizable, but we cannot exclude that it is entirely due to our systematic errors. A more computer intensive study should be performed to pin down differences, as well as L and k dependences. Anyhow, in our opinion, we have shown evidence that the TEK model will continue to be a powerful tool to explore the dynamics of large N Yang-Mills theory.

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